STATISTICAL INDICES

What They Are & How to Communicate Them

Matt Savage and Eric Gasper
PQ Systems

The webinar will begin shortly...
## Alternative view #1

<table>
<thead>
<tr>
<th>Date</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/18</td>
<td>34.64</td>
</tr>
<tr>
<td>1/2/18</td>
<td>2.245</td>
</tr>
<tr>
<td>1/3/18</td>
<td>20</td>
</tr>
<tr>
<td>1/4/18</td>
<td>10.45</td>
</tr>
<tr>
<td>1/5/18</td>
<td>0.015</td>
</tr>
<tr>
<td>1/6/18</td>
<td>36.1</td>
</tr>
<tr>
<td>1/7/18</td>
<td>34.64</td>
</tr>
<tr>
<td>1/8/18</td>
<td>54</td>
</tr>
<tr>
<td>1/9/18</td>
<td>17.8</td>
</tr>
<tr>
<td>1/10/18</td>
<td>15.24</td>
</tr>
<tr>
<td>1/11/18</td>
<td>34.64</td>
</tr>
<tr>
<td>1/12/18</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Alternative view #3

Individuals
UCL = 51.1214, Mean = 34.2715, LCL = -22.5784 (not shown), **
Alternative view #4
### Summary

#### Statistics

<table>
<thead>
<tr>
<th>Characteristic Name</th>
<th>Mean</th>
<th>Sigma</th>
<th>Max. Value</th>
<th>Min. Value</th>
<th>Cpk</th>
<th>Percent Out-of-Spec</th>
<th>Ppk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Data</td>
<td>14.2715</td>
<td>13.697</td>
<td>54.000</td>
<td>0.015</td>
<td>0.83</td>
<td>4.2%</td>
<td>0.75</td>
</tr>
</tbody>
</table>
### Summary

**Run Chart**
- UCL = 51.12 ± 4
- Mean = 14.27

**Statistics**

<table>
<thead>
<tr>
<th>Characteristic Name</th>
<th>Mean</th>
<th>Sigma</th>
<th>Max Value</th>
<th>Min Value</th>
<th>Cpk</th>
<th>Out-of-Spec</th>
<th>Ppk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Data</td>
<td>14.2715</td>
<td>13.697</td>
<td>54.000</td>
<td>0.015</td>
<td>0.83</td>
<td>4.2%</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Statistical Indices

- Ppk
- R-bar
- Chi-Square
- P-value
- Variance
- Poisson
- WTH
- EWMA
- LSL
- Type I Error
- OOS
- P-bar
- Anderson-Darling
- Null hypothesis
- Cr
- Cpm
- Sigma
- SQRT
- Six Sigma
- Central Limit Theorem
- Target
- ULS
- Null hypothesis
- Cpk
- Probability
- OOC
- X-bar
- Six Sigma
- Central Limit Theorem
- Target
- U-bar
- PPL
- Cpm
- Unimodal
- Cpl
- A2
- d2
- Sigma
- R-bar
- UCL
- OOC
- Variance
- Response variable
- Poisson
- Type I Error
- X-bar
- Normality
- Cpm
- Cpl
Formulas

\[ C_{pm} = \frac{USL - LSL}{6\hat{\sigma}_{C_{pm}}} \]

\[ \hat{\sigma}_{C_{pm}} = \sqrt{\frac{\sum (X_i - T)^2}{n-1}} \]
Purpose of:

• Specifications
• Mean
• Standard Deviation
Purpose of:

• Control charts
• Capability analysis
Specs limits come from:

- Customers
- Engineering
- Management
- Regulatory agencies
- Others

LSL
TSL
USL
Control limits come from:

- Customers
- Engineering
- Management
- Regulatory agencies
- Others
Control limits come from the process
Control limits reflect actual process variation

Specification limits reflect allowable process variation

Voice of the Process

Voice of the Customer
So …

• … can adding specifications lines to a control chart be harmful?
• … is there benefit by adding specifications lines to a control chart?
Sample size = 1

Specifications: Upper = 110.8921, Target = 99.871, Lower = 88.8498
UCL = 110.8921, Mean = 99.8710, LCL = 88.8498, **
Sample size = 1
Sample size = 8
Sample size = 8

Statistics

Basic Statistics
- 200 data values
- Maximum: 109.9450
- Mean: 99.871
- Minimum: 90.7865
- Sigma of the individuals: 3.522
- Within 1 Sigma (e): 69.5000%
- Within 2 Sigma (e): 95.0000%
- Within 3 Sigma (e): 100.0000%

Subgroup Statistics
- SS = 8
- Estimated Sigma: 3.571

Specifications
- Upper Spec: 103.6581
- Target Spec: 99.871
- Lower Spec: 96.0839

Out-of-spec
- Above: 15.0000%
- Below: 14.0000%
- Total: 29.0000%

Footnotes
- (e) = Uses Estimated sigma

29%
So …

• … can adding specification lines to a control chart be harmful even when the subgroup size is 1?
• … are there benefits to adding specification lines to a control chart?
100% In spec

Specifications: Upper = 115, Target = 100, Lower = 85
100% In spec, Cpk = 1.0
100% In spec, Cpk = 1.36
Specs on an Individuals chart

Individuals
Set 1: UCL = 411.5283, Mean = 395.6730, LCL = 379.8177, from: 1 to: 126
### Statistics

<table>
<thead>
<tr>
<th>Basic Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 data values</td>
</tr>
<tr>
<td>Mean 395.94</td>
</tr>
<tr>
<td>Sigma of the individuals 5.12</td>
</tr>
<tr>
<td>Dpm (e) 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subgroup Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>mR = 2</td>
</tr>
<tr>
<td>Estimated Sigma 5.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pp 1.63</td>
</tr>
<tr>
<td>Ppk 1.57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capability Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp 1.58</td>
</tr>
<tr>
<td>Cpk 1.52</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Spec     420</td>
</tr>
<tr>
<td>Target Spec</td>
</tr>
<tr>
<td>Lower Spec     370</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Out-of-spec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above 0.00%</td>
</tr>
<tr>
<td>Below 0.00%</td>
</tr>
<tr>
<td>Total 0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Footnotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e) = Uses Estimated sigma</td>
</tr>
</tbody>
</table>
Process change

Individuals
Set 1: UCL = 411.5283, Mean = 395.6730, LCL = 379.8177, from: 1 to 126

New Raw Materials
Recommendations

• Keep control charts in their ‘purest’ form
• Show spec information – not spec lines
• Add % out-of-spec
• Don’t make me think
Minimal Statistical Indices

![Diagram with mean (μ), lower specification limit (LSL), and upper specification limit (USL)]
Customers & You

- In-Control, Stable, Predictable
- Out-of-Control, Not Stable, Not Predictable
- Capable
- Not Capable
Capable - Not Capable
Cpk
Ppk
Cpm
Types of Capability Indices

- \( Cp \)
- \( Cr \)
- \( Cpk \)
- \( Cpu \)
- \( Cpl \)
- \( Pp \)
- \( Pr \)
- \( Ppk \)
- \( Ppu \)
- \( Ppl \)
- \( Cpm \)
Capability Explanation
Ranked by Out-of-Control
Ranked by % Out-of-Spec
“... and go on till you come to the end; then stop.”

Alice's Adventures in Wonderland - Lewis Carroll

Eric Gasper
EricG@PQSystems.com

Matthew Savage
Matt@PQSystems.com