Welcome!

The "Control Chart and Capability 101" webinar will begin shortly

Presented by

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PQ Systems
Webinar Goals
Webinar Goals

Central tendency

Variability
Webinar Goals

Data
Webinar Goals
What is Capability Analysis?
Requirement

Y

USL

X

Y

LSL

USL

X
Requirement
Control limits reflect **actual** process variation
Voice of the Process

Specification limits reflect **allowable** process variation
Voice of the Customer
Four States of Quality

- In control and capable
- Out-of-control, but capable
- In control but not capable
- Out-of-control and not capable
In Control & Not Capable
Out-of-Control & Not Capable
Out-of-Control, but Capable
Control Charts for Count Data

- **p-chart**
- **np-chart**
- **c-chart**
- **u-chart**

Nonconforming or Defective

Nonconformities or Defects
Control Charts – Measurement Data

- X – Individuals
- MA – Moving average
- X-bar
- MR – Moving range
- R – Range
- S – Sigma
Control charts are used to:

- Determine central tendency
- Monitor variability
- Spot trends
- To determine stability and therefore predictability
Purpose of Capability Analysis
Control Chart Intro / Refresher

- What is it?
- What does it look like?
- When is it used?
- How is it made?
What is it?

*Average / Individuals and variability*

- Monitors how a system changes over time
- Monitors averages to show trends and shifts
- Monitors range to indicate changes in variation
- Used for measurement data
Characteristic: Gap of Dimension A

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X-bar: 0.700, 0.770, 0.760, 0.680, 0.750, 0.730, 0.730, 0.730, 0.720, 0.720, 0.720, 0.710, 0.710, 0.710, 0.710, 0.710

Range: 0.200, 0.200, 0.150, 0.200, 0.250, 0.150, 0.200, 0.200, 0.250, 0.200, 0.200, 0.200, 0.200, 0.200, 0.200, 0.200

USL = 0.90, LSL = 0.50
UCL = 0.819, Mean = 0.716, LCL = 0.613, **

Range

UCL = 0.376, Mean = 0.178, LCL (none), **
When is it used?

Answer "yes" to the following:

– Do you need to assess the variability in the system?
– Can the data be collected or does a collection of data already exist?
– Is the time order of the data preserved?
– Is the data in variables format?
– Is the data collected in an appropriate subgroups size?
How is it made?

- Assumes data has been collected
  - Ideally 25 or more data points.
- Any unusual occurrences observed during data collection should have been noted.
How is it made?

1. Complete the header information.
2. Record the data.
3. Calculate the statistics for each subgroup.
4. Calculate the overall averages.
5. Calculate the control limits.
6. Scale the control chart.
7. Draw the average line and control limits.
8. Plot the values on the control chart.
9. Interpret the control chart.
How is it made?

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- *Basic rules for interpretation*
  - Any point lying outside the control limits.
  - Run of seven points:
    » Seven or more points in a row above or below the center line.
    » Seven or more points in a row going in one direction, up or down.
  - Any non-random pattern, including the following typical cases:
    » Too close to the average.
    » Too far from the average.
    » Cycles.
Any point beyond the control limits
Seven consecutive below the mean
Seven consecutive decreasing
Too far from the average, cycles, ...
Tip: Use enough data

• How much data is necessary for control limits on a control chart?

• “An X-mr chart can be made with as few as four original values without an undue risk of a false alarm.” Donald J. Wheeler, Ph.D.

• Data beyond a control limit? Investigate it — even with only a few samples.
Purpose of a Control Chart
Common and Special Cause Variation

- Process Control
- Out of Control (Special Causes Present)
- In Control (Special Causes Eliminated)

Size → Time
Types of Capability Indices

- Cr
- Cp
- Cpk
- Cpu
- Cpl
- Cpm
- Pr
- Pp
- Ppk
- Ppu
- Ppl
Symbols and Formulas
Symbols and Formulas

- **Sigma-e** = estimated sigma
  \[ \hat{\sigma}_e = \frac{R}{d_2} \]

- **Sigma-i** = standard deviation of the individual readings
  (Excel uses the STDEV function)
  \[ \hat{\sigma}_i = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}} \]
Symbols and Formulas – Cp

\[ Cp = \frac{USL - LSL}{6 \times \hat{\sigma}_e} = \frac{\text{Voice of the Customer}}{\text{Voice of the Process}} \]
Symbols and Formulas – \( C_P \)

\[
C_P = \frac{USL - LSL}{6 * \hat{\sigma}_e}
\]
Symbols and Formulas - Cp

- \( Cp = 1.0 \)

- The allowable spread is the same as the actual spread
Cp > 1

- The allowable spread is wider than actual spread
Symbols and Formulas - \( C_p \)

- \( C_p > 1 \)

- The vehicle can fit
Symbols and Formulas – Cp

- $C_p < 1.0$
- The space is too narrow
Symbols and Formulas

\[
C_{pu} = \frac{(USL - \bar{X})}{(3 \cdot \hat{\sigma}_e)}
\]

\[
C_{pl} = \frac{(\bar{X} - LSL)}{(3 \cdot \hat{\sigma}_e)}
\]

\[
C_{pk} = \text{Smaller of } C_{pu} \text{ and } C_{pl}
\]
Key Assumptions

- A control chart is completed
- The control chart shows no special causes
- One or both specifications exist
- The distribution of the data is bell-shaped
How Capability Analysis is Performed

- Calculate the mean and the standard deviation
- Sketch the distribution & specification line(s)
- Calculate Z-values
- Determine expected percent out-of-spec
- Calculate the Cp, Cpk and/or Pp, Ppk
How Capability Analysis is Performed

- Sketch the distribution (optional, but helpful)

\[ \bar{X} = 10.0 \]
\[ \bar{R} = 5.81 \]
USL = 15.8
LSL = 5.73
\[ n = 5 \]
How Capability Analysis is Performed

- Calculate the standard deviation

Estimated sigma = $\frac{\bar{R}}{d_2}$

Est. sigma = $\frac{5.81}{2.326} \approx 2.50$

3 x sigma = 7.50

Mean +/- 3sigma = 17.50, 2.50

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How Capability Analysis is Performed

- Sketch the distribution (optional, but helpful)

\[
\bar{X} = 10.0 \\
\bar{R} = 5.81 \\
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How Capability Analysis is Performed

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\text{USL} = 15.8 \\
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\]
How Capability Analysis is Performed

- Sketch the distribution (optional, but helpful)

![Diagram showing process capability analysis]

- $\bar{X} = 10.0$
- $\bar{R} = 5.81$
- USL = 15.8
- LSL = 5.73
- $n = 5$
How Capability Analysis is Performed

- Calculate $Z_{upper}$ and $Z_{lower}$

\[
Z_{upper} = \frac{(USL - Mean)}{\sigma} = \frac{(15.8 - 10.0)}{2.50} = 2.32
\]

\[
Z_{lower} = \frac{(Mean - LSL)}{\sigma} = \frac{(10 - 5.73)}{2.50} = \approx 1.71
\]
## Standard Normal Table

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How Capability Analysis is Performed

\[ Z_{upper} = 2.32 \rightarrow 0.0102 \rightarrow 1.02\% \]

\[ Z_{lower} = 1.71 \rightarrow 0.0436 \rightarrow 4.36\% \]

We expect 1.02% + 4.36% of data to be outside the specification limits.
How Capability Analysis is Performed

- $C_p = \frac{\text{spec spread}}{\text{process spread}}$
- $C_p = \frac{(15.8 - 5.73)}{(6 * 2.50)}$
- $C_p = 0.67$

- $C_{pk} = \frac{\text{Smallest Z value}}{3}$
- $C_{pk} = \frac{1.71}{3}$
- $C_{pk} = 0.57$
Getting the Most from Capability Analysis

\[ C_{pm} = \frac{USL - LSL}{6 \hat{\sigma}_{C_{pm}}} \]

\[ \hat{\sigma}_{C_{pm}} = \sqrt{\frac{\sum (X_i - T)^2}{n - 1}} \]
Average is at the Target Value

\[ C_p = 0.5 \]
\[ C_{pk} = 0.5 \]
\[ C_{pm} = 0.5 \]
Average is Off Target

Average is Off Target

\[ C_p = 2.0 \]
\[ C_{pk} = 1.0 \]
\[ C_{pm} = 0.65 \]
Average is at the Target Value

\[ C_p = 1.5 \]
\[ C_{pk} = 1.5 \]
\[ C_{pm} = 1.5 \]
Final Thoughts

• Control charts are designed to provide stability information

• Capability analysis allows you to compare different processes

• Capability analysis brings together the process limits (+/- 3 sigma) and specification limits
PQ Systems Software

SQC pack  GAGE pack

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