

# STATISTICAL INDICES

What They Are &  
How to Communicate Them

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PQ Systems



*The webinar will begin shortly...*





# Data

2.245 20 12.85 7.657 13.61 28.77 18.243  
5.08 8.13 6.955 5.08 34.64  
1.394 1.26 17.998  
10.45 10.45 6.357  
.015 .196  
36.10 2.536 .092  
34.64 .08 .049 50.4 .08  
4.698 18.243  
54 7.657 15.24 7.657  
15.24 17.998  
17.8 2.245 11.245  
34.64 11.245 17.8 12.85

# Data

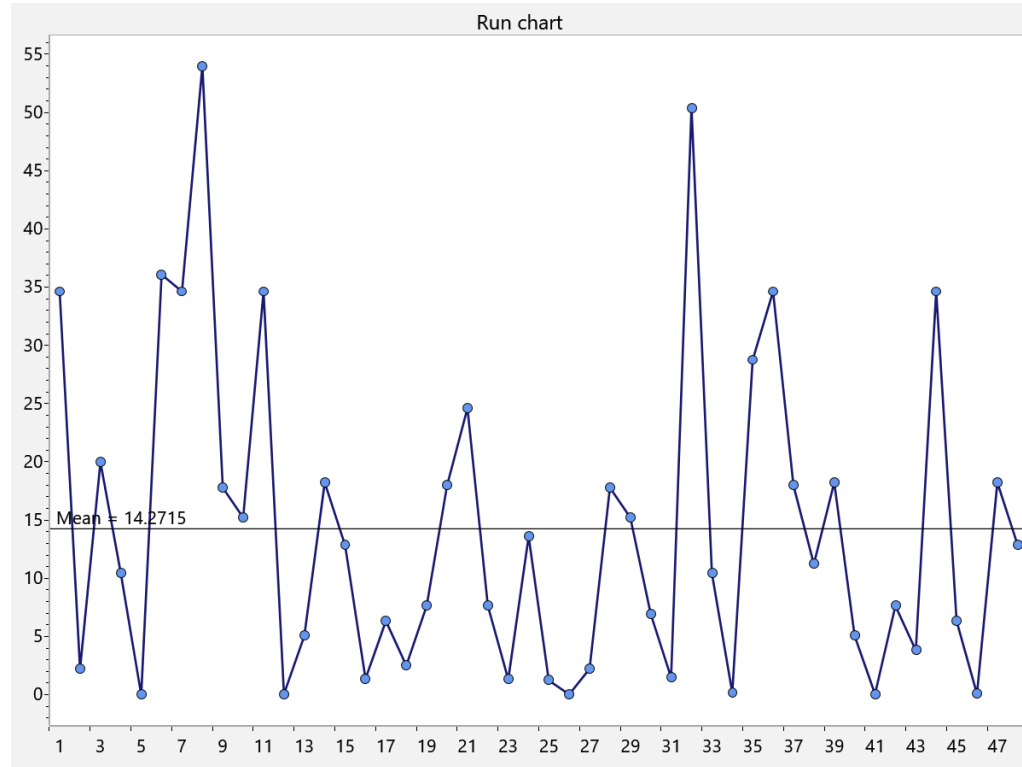
2.245 20 12.85 7.657 13.61 28.77 18.243  
5.08 8.13 6.955 5.08 34.64  
1.394 1.26 17.998  
10.45 10.45 6.357  
**.015** 6.357  
36.10 2.536 .196 .092  
34.64 .08 .049 50.4 .08 18.243  
4.698 15.24 7.657  
**54** 17.998 7.657  
15.24 2.245 11.245  
17.8 34.64 11.245 17.8 12.85

# Alternative view #1

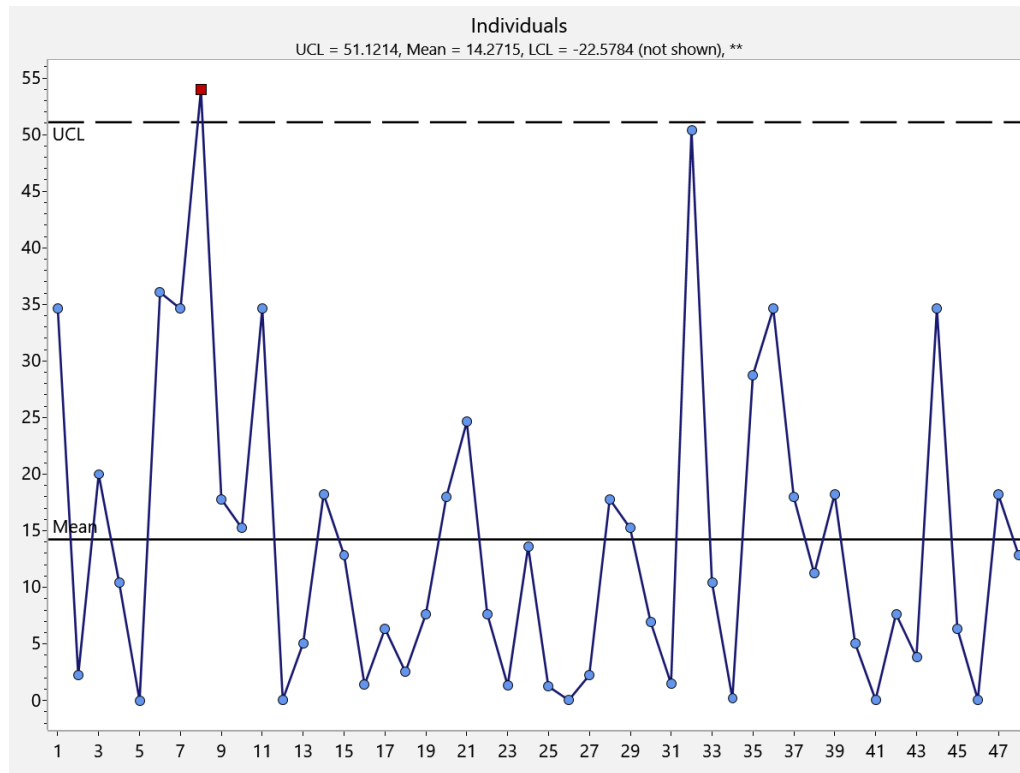


Date	Values
1/1/18	34.64
1/2/18	2.245
1/3/18	20
1/4/18	10.45
1/5/18	0.015
1/6/18	36.1
1/7/18	34.64
1/8/18	54
1/9/18	17.8
1/10/18	15.24
1/11/18	34.64
1/12/18	0.08

# Alternative view #2

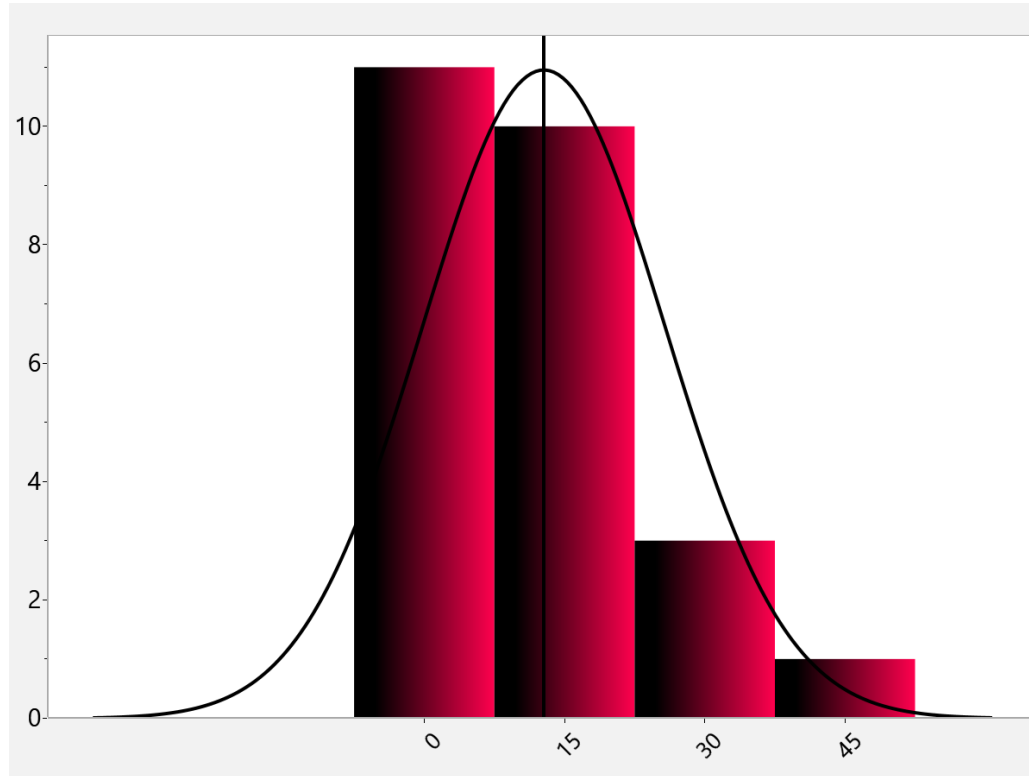


# Alternative view #3





# Alternative view #4



# Alternative view #5



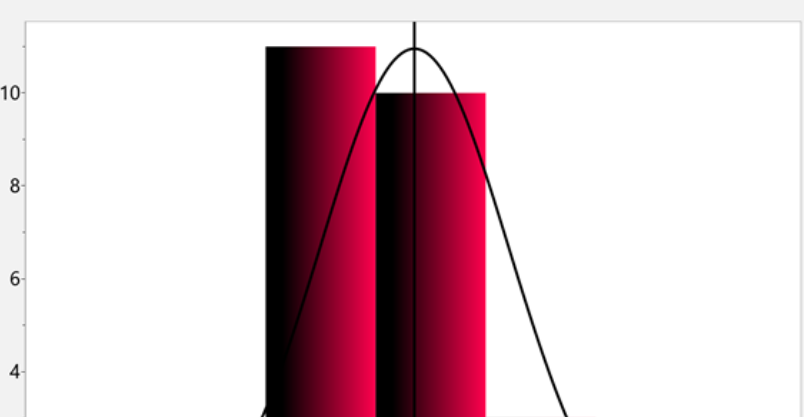
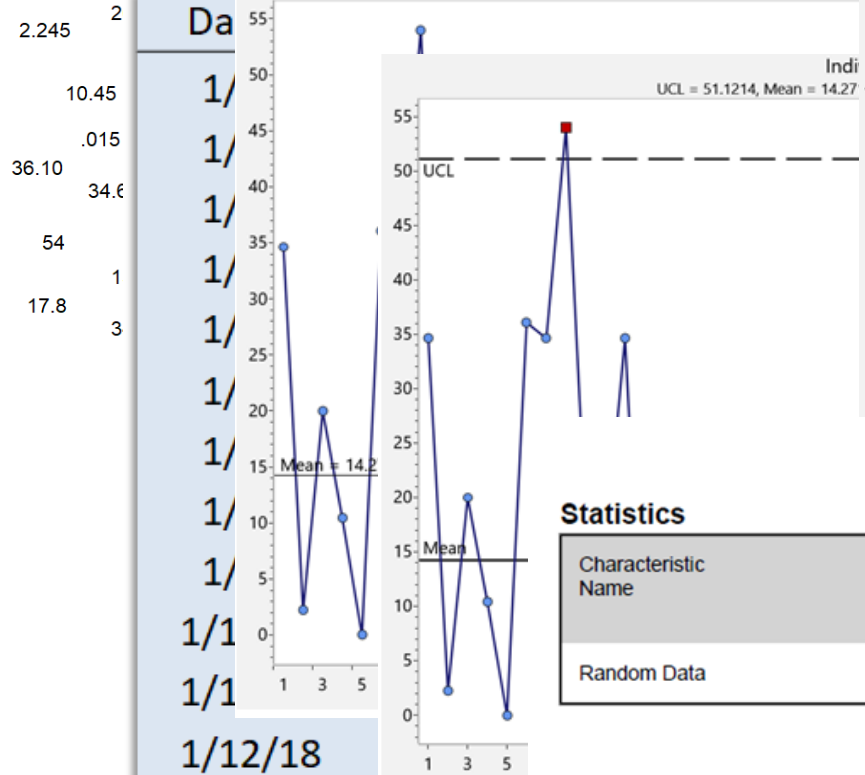
## Summary

### Statistics

Characteristic Name	Mean	Sigma	Max. Value	Min. Value	Cpk	Percent Out-of-Spec	Ppk
Random Data	14.2715	13.697	54.000	0.015	0.83	4.2%	0.75

34.64 18.243 24.63 1.34 1.5 34.64 3.84

Run chart



Summary

Statistics

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Random Data	14.2715	13.697	54.000	0.015	0.83	4.2%	0.75

1/12/18

# Statistical Indices

Ppk Probability SQRT Chi-Square P-value Cr  
A<sub>2</sub> p-bar Anderson-Darling Six Sigma Cpm  
Null hypothesis Cpk OOS Central Limit Theorem WTH  
Variance CL Target  
d<sub>2</sub> Sigma R-bar UCL Response variable PPL U-bar  
LSL OOC Poisson Cpm  
P<sub>z</sub> Variance Unimodal  
R-bar Normality EWMA Type I Error  
Ppu X-bar Cpl

Cpk

Ppk

Cp

Pp

Cr

Pr

Cpu

Ppu

Cpl

Ppl

Cpm

Pp

# Formulas



$$C_{pm} = \frac{USL - LSL}{6 \hat{\sigma}_{Cpm}}$$

$$\hat{\sigma}_{Cpm} = \sqrt{\frac{\sum (X_i - T)^2}{n - 1}}$$

## Purpose of:

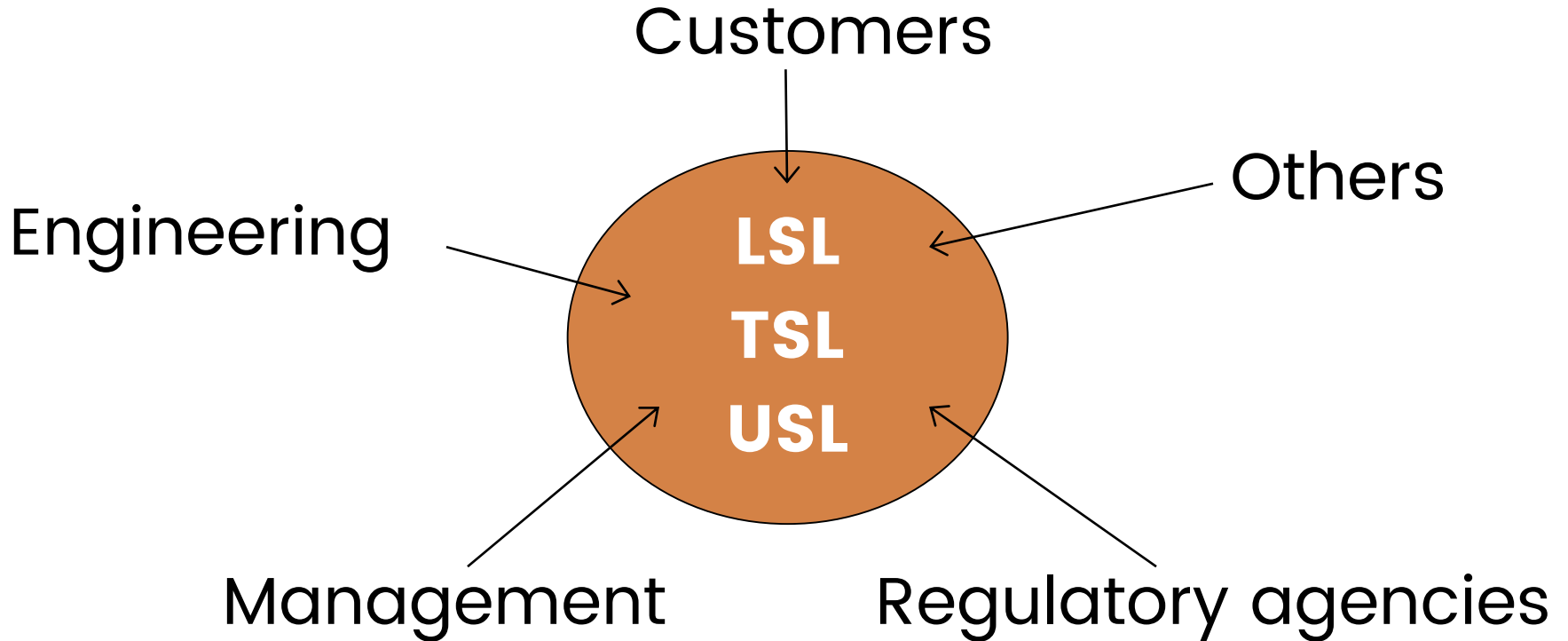
- Specifications
- Mean
- Standard Deviation

## Purpose of:

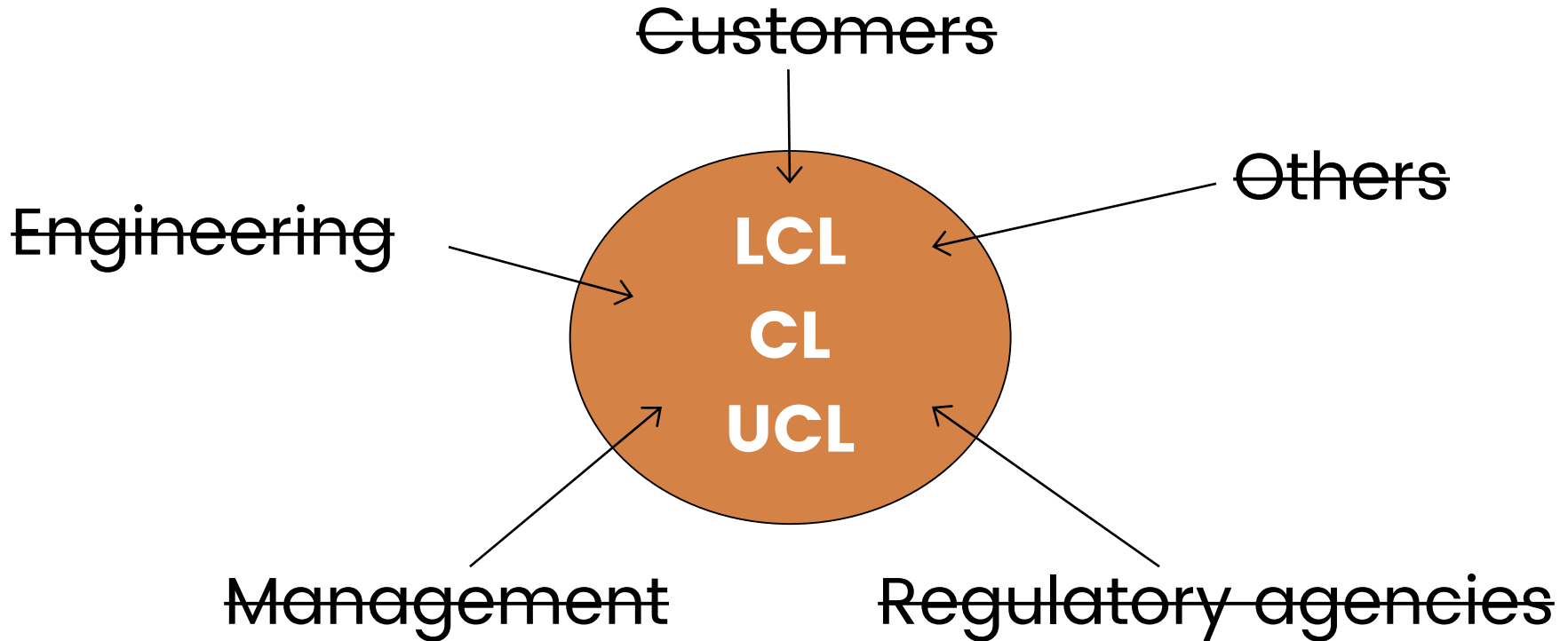
- Control charts
- Capability analysis



# Specs limits come from:



# Control limits come from:

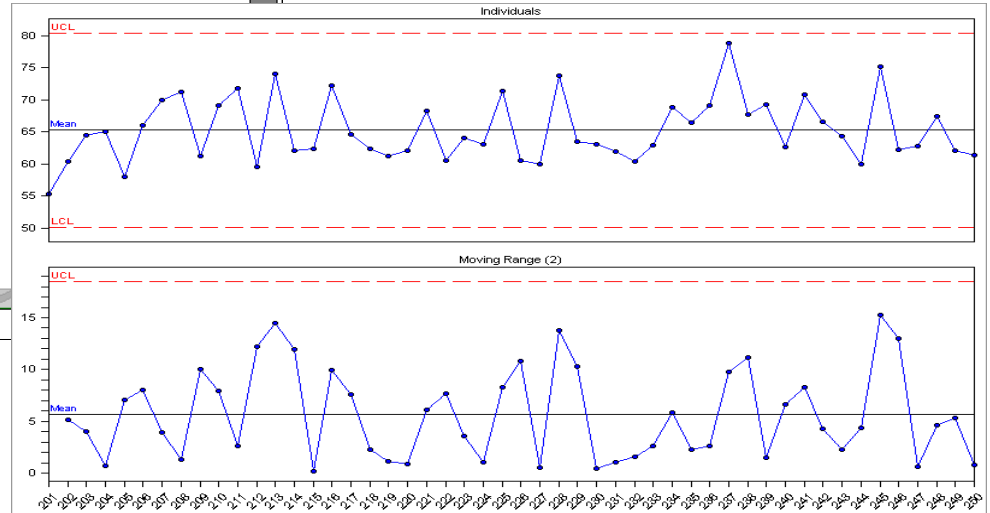
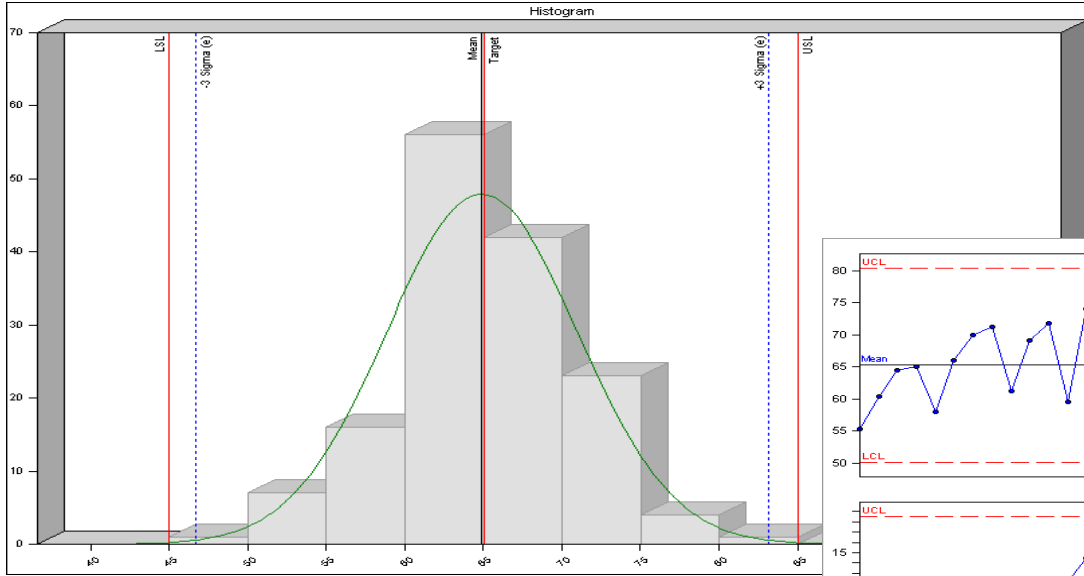


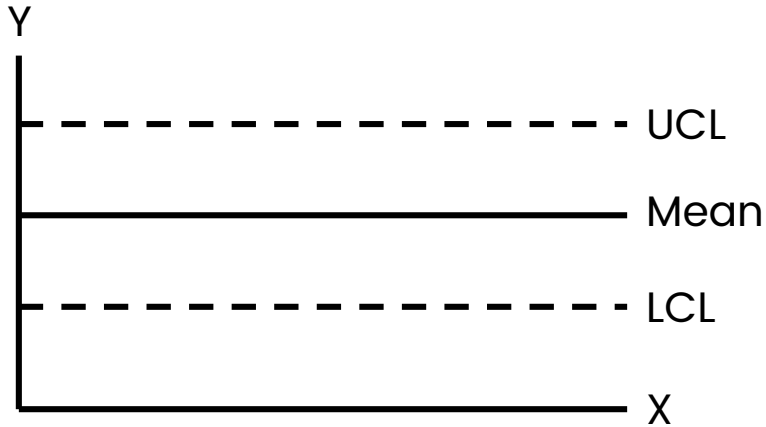
Control limits come from the process

Control limits come from the process

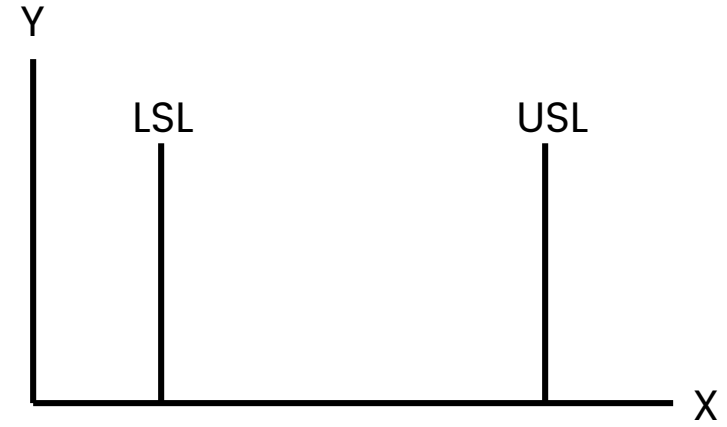
Control limits come from the process







$\neq$



Control limits reflect  
**actual** process variation

Voice of the **Process**

Specification limits reflect  
**allowable** process variation

Voice of the **Customer**

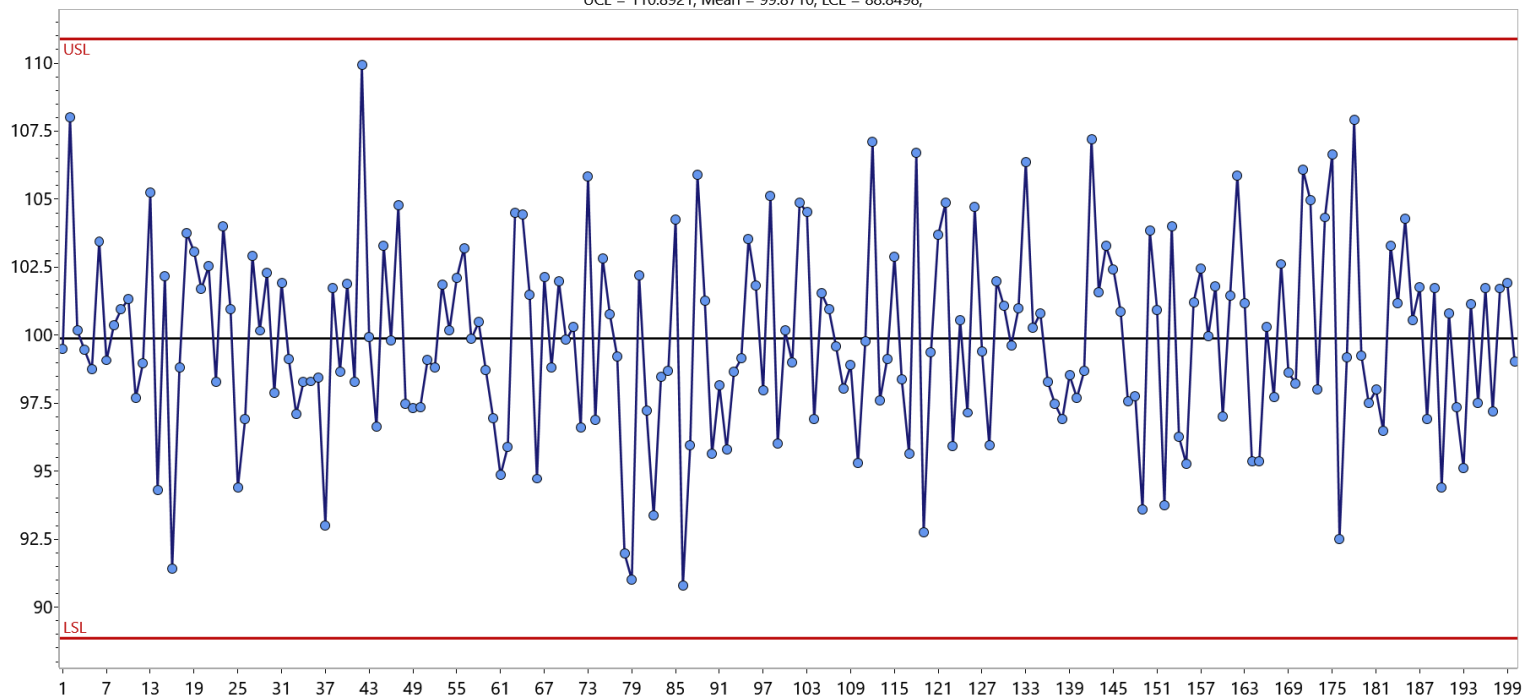
So ...

- ... can adding specifications lines to a control chart be harmful?
- ... is there benefit by adding specifications lines to a control chart?

# Sample size = 1

POWERED BY  SOC  
pack

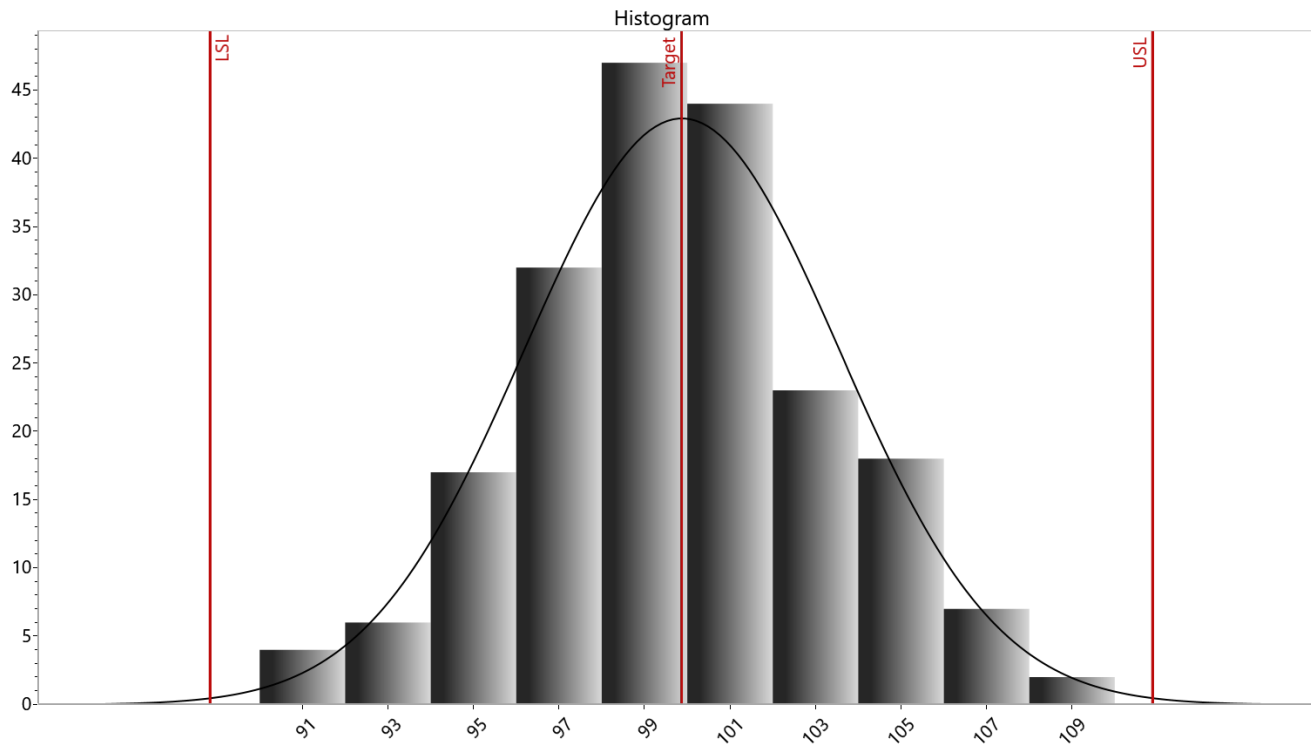
Specifications: Upper = 110.8921, Target = 99.871, Lower = 88.8498  
UCL = 110.8921, Mean = 99.8710, LCL = 88.8498, \*\*





# Sample size = 1

POWERED BY  **SOC**  
pack



## Statistics

Basic Statistics	
200 data values	
Maximum	109.9450
Mean	99.871
Minimum	90.7865
Sigma of the individuals	3.522
Within 1 Sigma (e)	71.000%
Within 2 Sigma (e)	96.000%
Within 3 Sigma (e)	100.000%

Subgroup Statistics	
mR = 2	
Estimated Sigma	3.674

Specifications	
Upper Spec	110.8921
Target Spec	99.871
Lower Spec	88.8498

Out-of-spec	
Above	0.000%
Below	0.000%
Total	0.000%

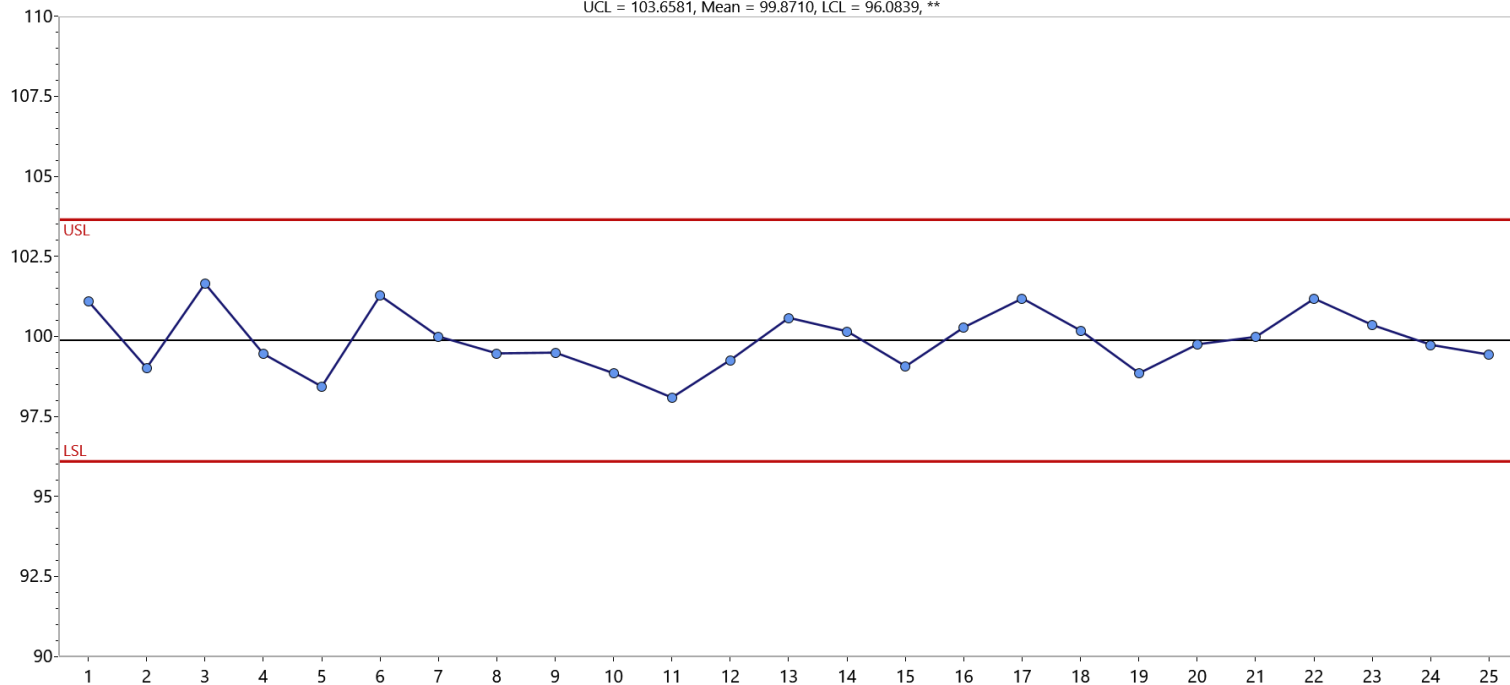
  

Footnotes	
(e) = Uses Estimated sigma	

# Sample size = 8

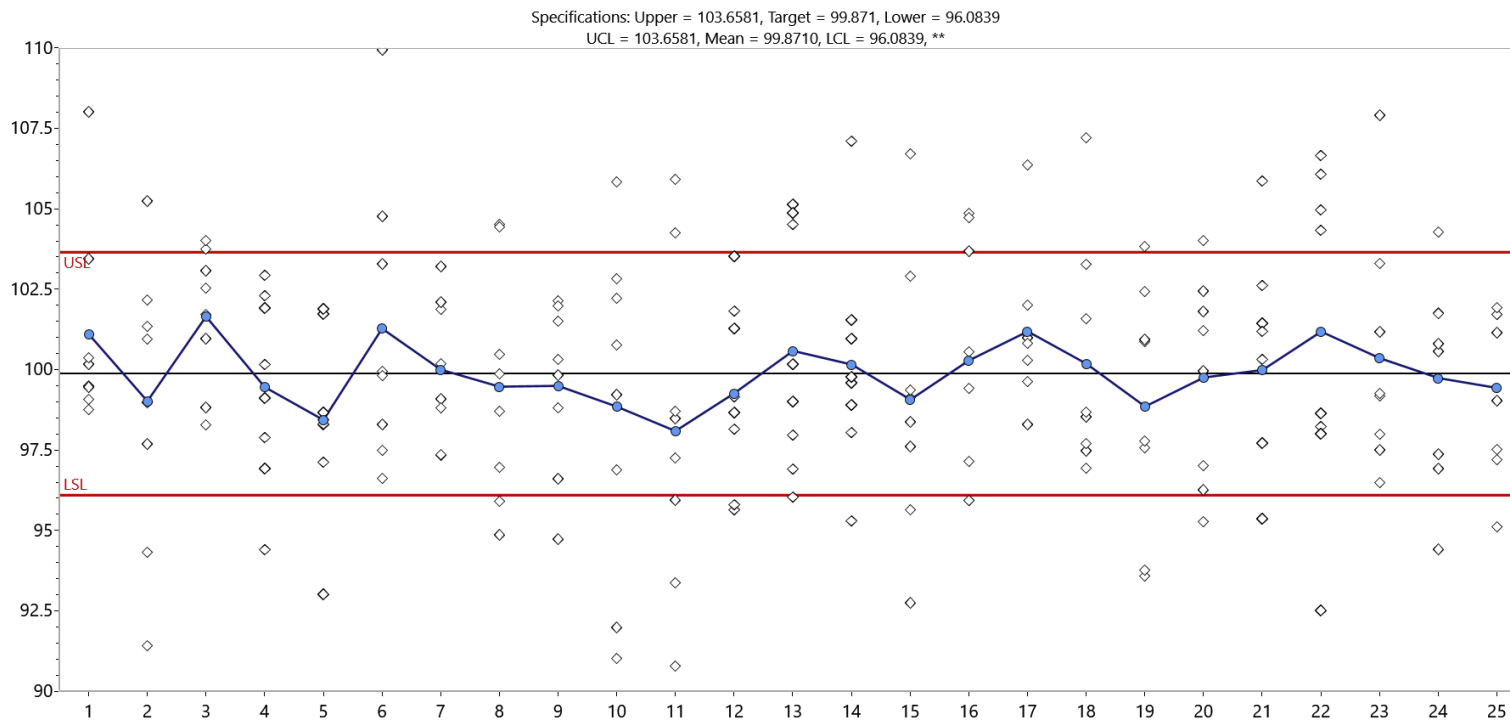
POWERED BY  SOC  
pack

Specifications: Upper = 103.6581, Target = 99.871, Lower = 96.0839  
UCL = 103.6581, Mean = 99.8710, LCL = 96.0839, \*\*



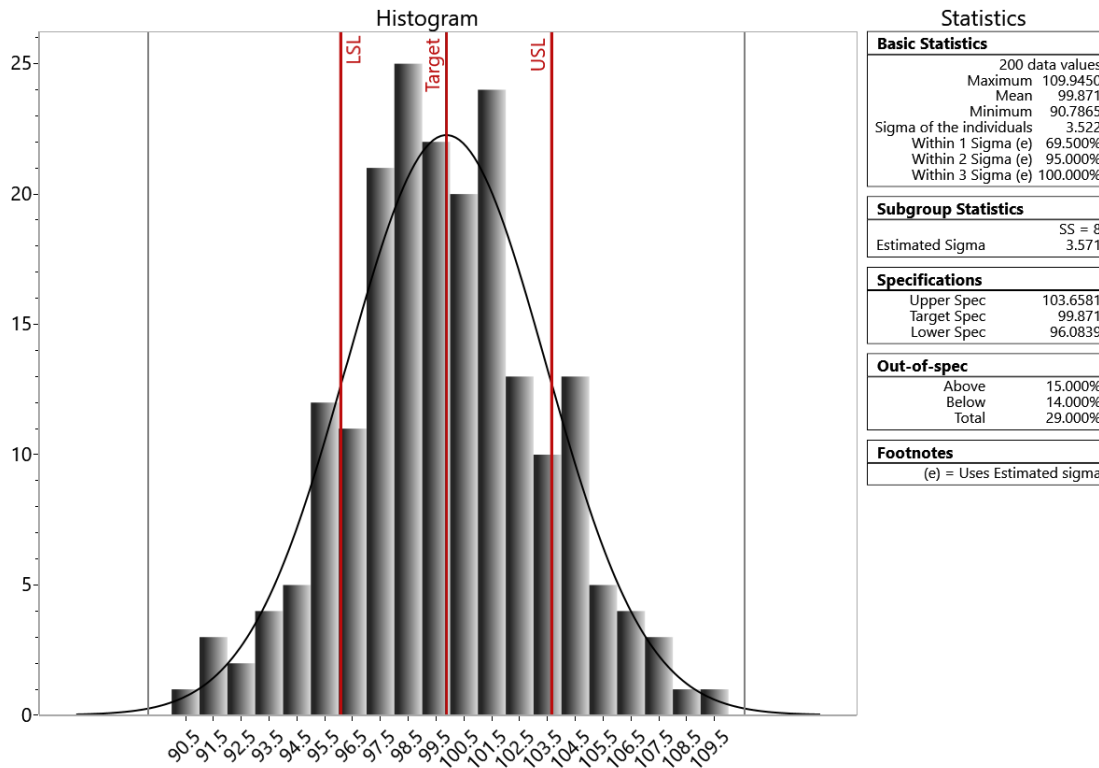
# Sample size = 8

POWERED BY  SOC  
pack



# Sample size = 8

POWERED BY  SOC  
pack



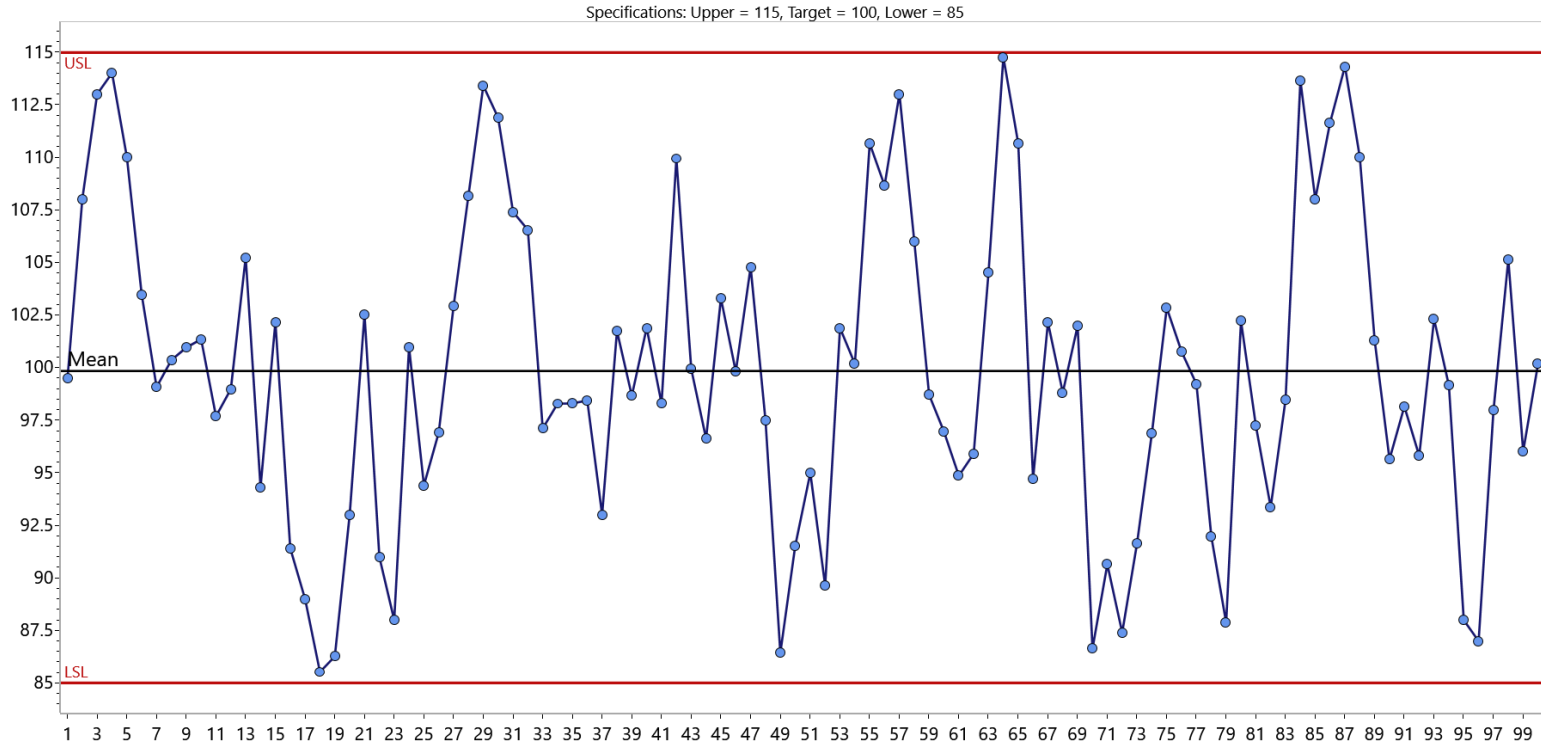
29%

So ...

- ... can adding specification lines to a control chart be harmful even when the subgroup size is 1?
- ... are there benefits to adding specification lines to a control chart?

# 100% In spec

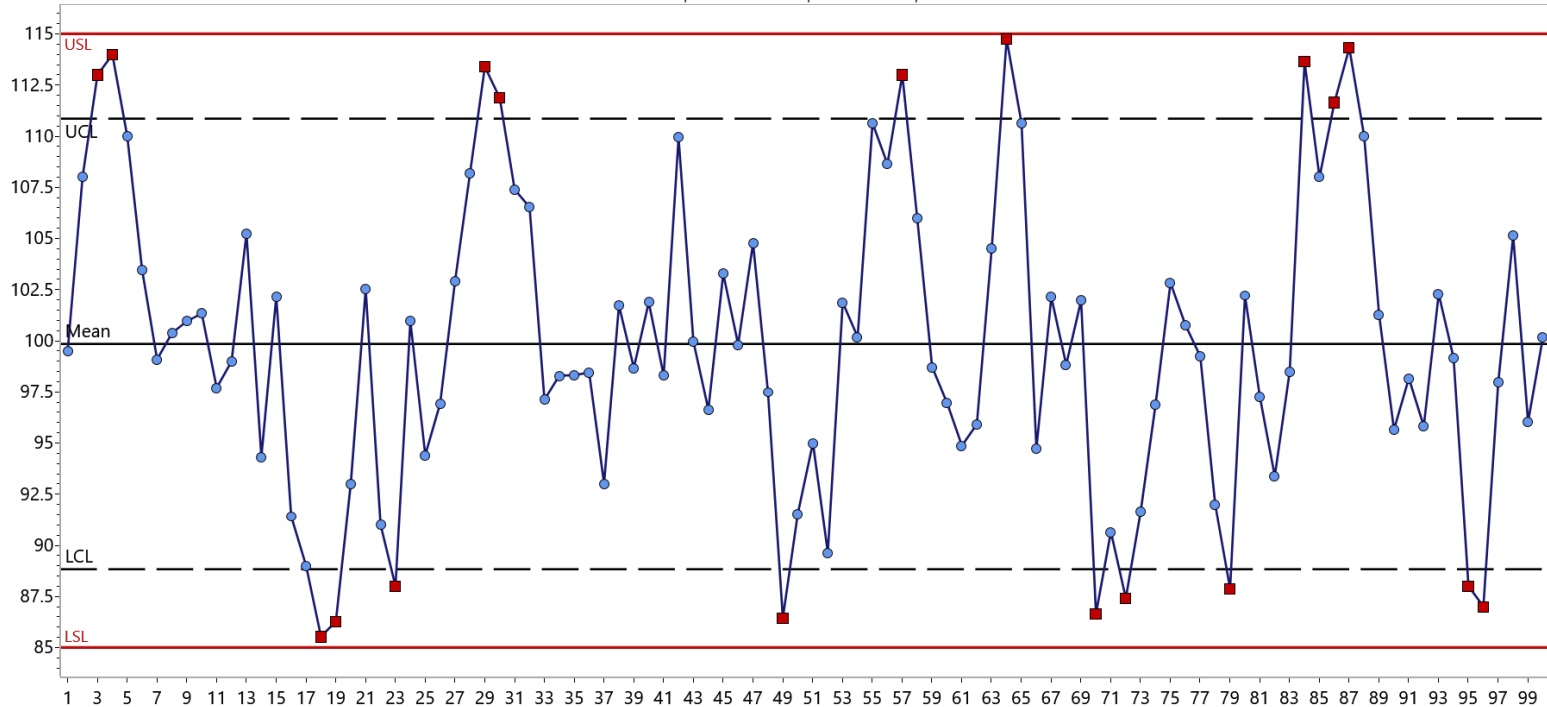
POWERED BY  SOC  
pack



# 100% In spec, Cpk = 1.0

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pack

Specifications: Upper = 115, Target = 100, Lower = 85  
Set 1: UCL = 110.8500, Mean = 99.8500, LCL = 88.8500, from: 1 to: 200

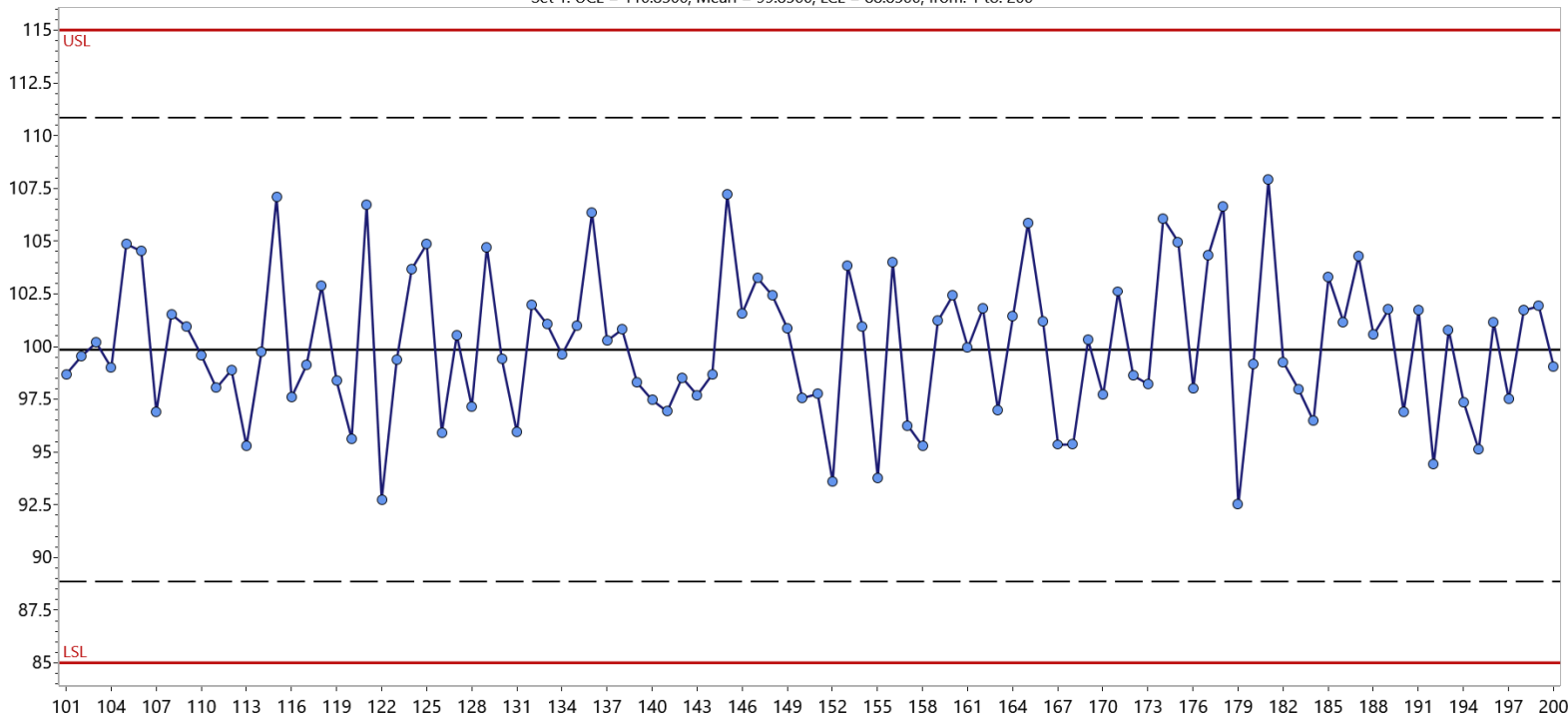


# 100% In spec, Cpk = 1.36



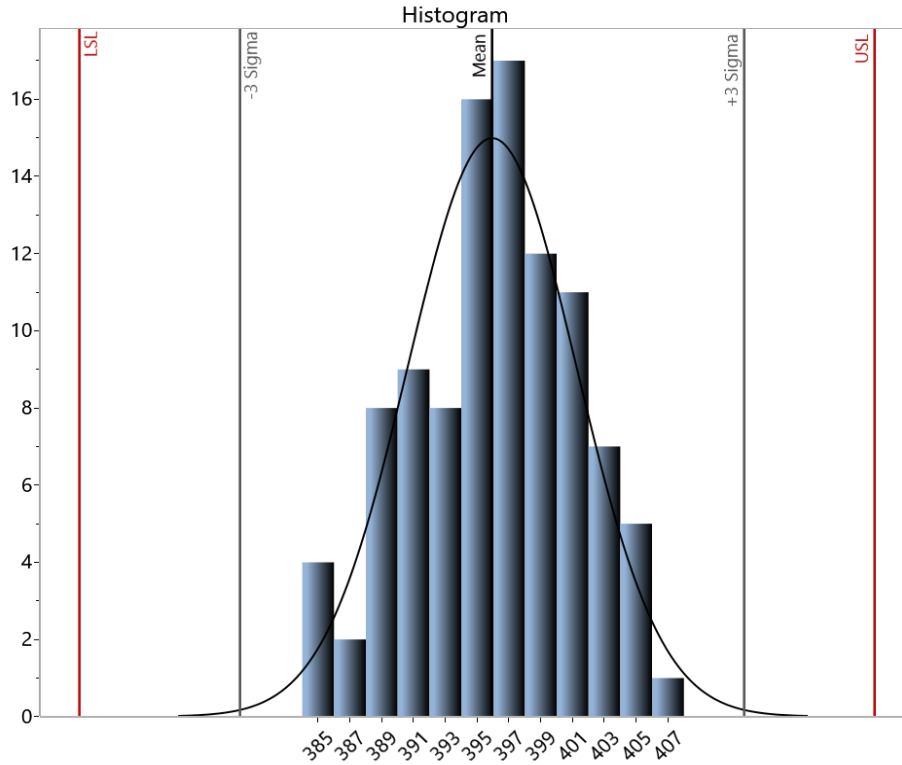
POWERED BY SOC pack

Specifications: Upper = 115, Target = 100, Lower = 85  
Set 1: UCL = 110.8500, Mean = 99.8500, LCL = 88.8500, from: 1 to: 200









### Statistics

Basic Statistics	
100 data values	
Mean	395.94
Sigma of the individuals	5.12
Dpm (e)	3

Subgroup Statistics	
Estimated Sigma	mR = 2 5.29

Performance Statistics	
Pp	1.63
Ppk	1.57

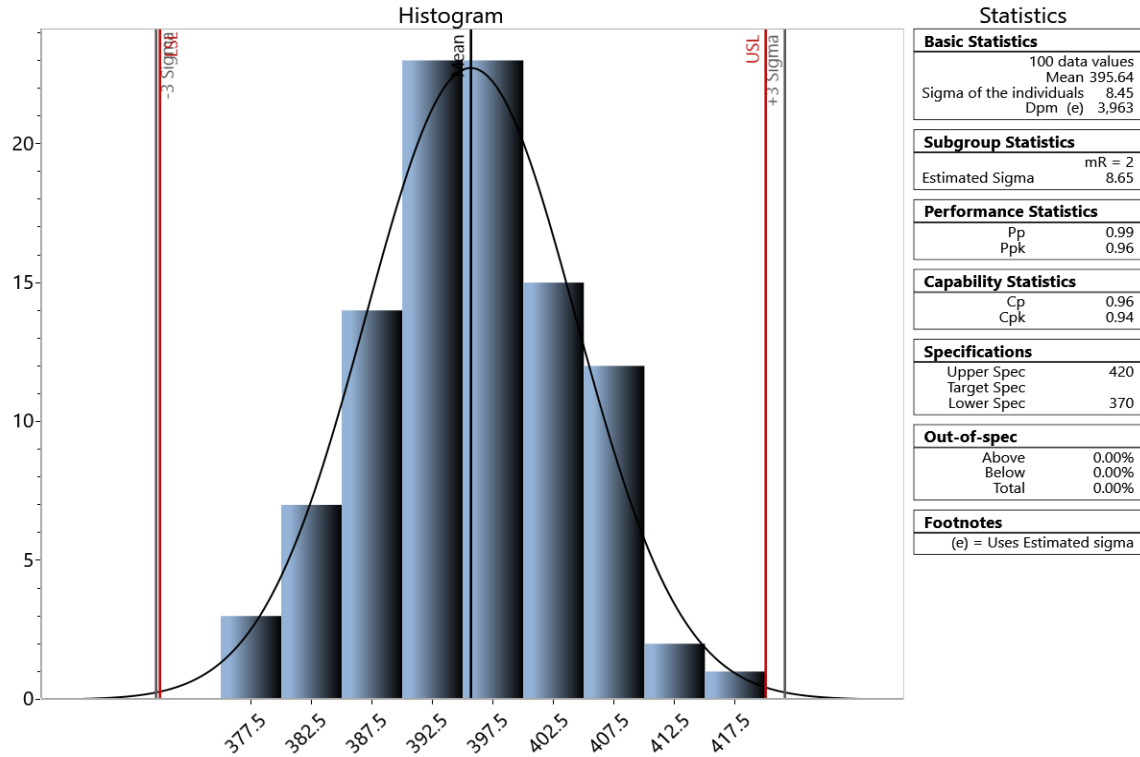
Capability Statistics	
Cp	1.58
Cpk	1.52

Specifications	
Upper Spec	420
Target Spec	
Lower Spec	370

Out-of-spec	
Above	0.00%
Below	0.00%
Total	0.00%

Footnotes	
(e)	= Uses Estimated sigma



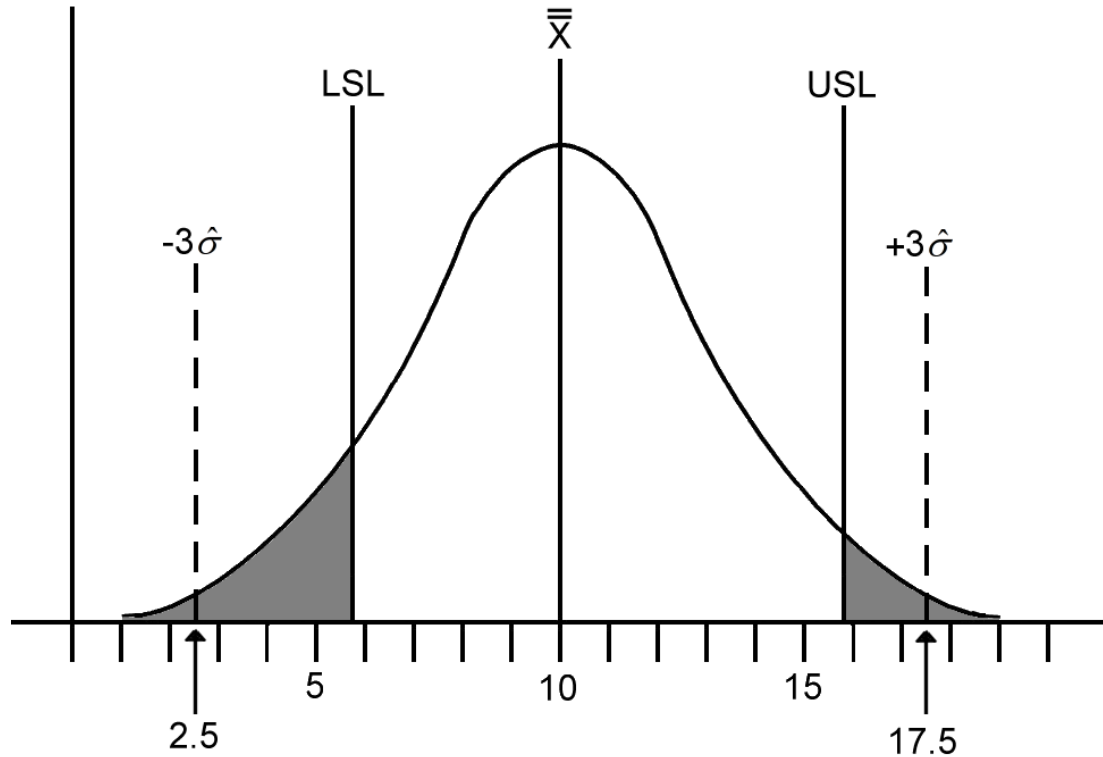


# Recommendations



- Keep control charts in their 'purest' form
- Show spec information – not spec lines
- Add % out-of-spec
- Don't make me think

# Minimal Statistical Indices



# Customer Requirements



# Customers & You



- In-Control, Stable, Predictable
- Out-of-Control, Not Stable, Not Predictable
- Capable
- Not Capable



Capable - Not Capable

Cpk

Ppk

Cpm

# Types of Capability Indices



- Cp
  - Cr
  - Cpk
  - Cpu
  - Cpl
- Pp
  - Pr
  - Ppk
  - Ppu
  - Ppl
- Cpm

# Capability Explanation



# Capability Explanation



# Ranked by Out-of-Control





# Ranked by % Out-of-Spec



**“... and go on till you come to the end; then stop.”**

*Alice's Adventures in Wonderland - Lewis Carroll*

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