An Alternative Method for Design of Experiments

This Six Sigma tool requires no statistics and can be performed while a process is running.

by Mark L. Crossley

Interest in Six Sigma continues to remain very high as organizations use its tools to improve their processes, products and services. One of the major tools utilized in the Six Sigma protocol is design of experiments (DOE). In nearly all applications of DOE we follow either the Western (i.e., traditional) or the Taguchi approach, with the former predominating.

In either case the objective of a designed experiment is to identify those inputs (factors) that influence an output (response) of a process or design. The effect of these factors can be ranked and analyzed to determine the level of statistical significance. The determination of statistical significance can be determined graphically by performing a normal probability plot (NOPP) or more precisely by performing an analysis of variance (ANOVA). The latter analysis is more complex and generally involves the use of statistical software.

One issue with these traditional DOE approaches is that they usually must be performed offline because some of the experimental runs can produce process results beyond process specification limits.

Know & Go

- Simplex Optimization, or simplex, is an alternative approach to traditional design of experiments (DOE) that offers the quality practitioner the ability to explore, through many experiments, the response space of their process. Process changes are made in small incremental amounts as the process is "tweaked" for enhanced performance.
- Simplex also offers a method to verify results using a minimal amount of computation. Complex analysis are not required and don't require the use of statistical software. Simplex is user-friendly in its approach.
- Simplex can employ graphical methods to track the progress of the experiments, especially with two or three experimental factors.

There are other methods by which we can accomplish the same end as with a traditional designed experiment. These introduce small changes that shouldn't adversely affect process results while zeroing in on optimal process parameters.

One such method is the concept of evolutionary operations (EVOP), which is discussed in the book *Evolutionary Operations* (George E. P. Box and Norman R. Draper, John Wiley and Sons, 1969). In an EVOP approach one augments the current operating conditions by small increments and migrates to a position or coordinate yielding an improvement. For a two-factor experiment the EVOP approach would start with four positions around the initial starting point.

Another method, Simplex Optimization, was introduced in *Sequential Simplex Optimization* (F.H. Walters, L. R. Parker, Jr., S.L. Morgan and S.N. Deming, CRC Press, 1991). In their discussion, however, there is no mention of statistical significance.

In this article I will discuss the concept of Simplex Optimization combined with the application of hypothesis testing as an alternative to traditional DOE with ANOVA. Little or no knowledge of statistics is required for the methodology.

In the following hypothetical case study we'll examine two factors. (In reality, any number of factors might be examined.) Three unique conditions or experiments (i.e., runs) are required to examine two factors using the simplex approach. In a traditional full-factorial experiment, four runs would be required. The figure at the upper left-hand side of the following page gives the number of runs as a function of the number of factors for both simplex and full-factorial DOE.

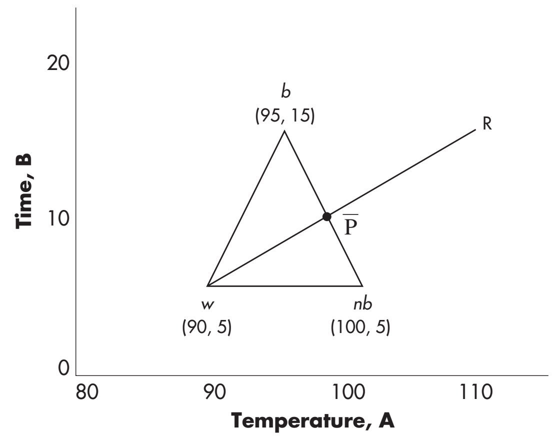
Simplex terminology

Simplex—A geometric figure defined by a number of coordinates equal to one more than the number of factors being exam-

Factors and Runs: Simplex Vs. Full-Factorial Experiments

Number of factors (F)	Simplex experiments (N)	Full-factorial experiments (N)
	N=F+1	$N=2^F$
1	2	2
2	3	4
3	4	8
4	5	16
5	6	32

Plot of Simplex 1



ined. An n-factor study will yield an n+1 number of vertexes.

Vertex—A corner of a simplex and one of the points that defines it. A two-factor study will have three corners for each simplex.

Face or hyperface—The part of a simplex that remains after removing one of the vertexes.

Centroid—The geometric center of a set of vertex coordinates. We'll be examining the centroid of a hyperface, referred to as $\overline{P}$.

Hypothetical case study

A manufacturing process requires that a lap-joint be made. There are several factors that could be examined to maximize the bond strength. These include the amount of adhesive, brand of adhesive, concentration of the adhesive, substrate smoothness, clamping pressure, temperature during curing and duration of clamping time. The latter two factors will be examined.

The current settings are 95° Fahrenheit for the temperature and 10 seconds for the clamping time. The number of factors is two; therefore, we'll need to define three sets of conditions for our initial experiment. These can be defined as coordinates. We'll perturb the normal temperature by ±5 degrees and the time by ±5 seconds. You could use any combination of temperature and time to arrive at the three coordinates. The temperature will be designated as factor A and the time as factor B (see the figure above right).

This two-factor simplex has three vertexes that form a triangle. The corner of each vertex locates a unique set of coordinates describing a unique experimental condition. As you can see, this is a two-dimensional simplex, having only an *x* and *y* dimension. A three-dimensional simplex (i.e., one with three factors) would have four vertexes. The resulting geometric figure, called a tetrahedron, would have four corners. It isn't necessary for the figures to be symmetric, but for three factors they must have some length, width and depth. (You might have more than three factors, but this will yield a simplex with four or more dimensions. These figures are referred to as "hypertetrahedra" and can't be visually conceptualized.)

We'll run the experiment at each of the three vertexes twice (for two replicates). The data will be recorded on the simplex 1 worksheet (see the figure at left), including the individual responses, the average and the standard deviation. The average responses will be ranked as "best" (*b*), "next best" (*nb*) and "worst" (*w*).

We'll now perform a hypothesis test to confirm that there's a statistically significant difference between *b* and *w*. If there's no difference, we must expand the degree of perturbation. No advice is given with respect to how much one should vary a given parameter, but remember that we want to improve by small increments.

H_o: $U_{best} = U_{worst}$ (no difference)

H_a: $U_{best} > U_{worst}$ ("best" is a real improvement over "worst")

The test statistic will be the calculated t-score:

n = The number of observations for each experimental run (two in this case).

S_p^2 = The pooled variance

Simplex 1 Worksheet

	Factor A	Factor B	Responses (bond strength)	Average $\overline{X}$	Standard deviation S	Rank
Coordinates	95	15	55, 54	54.5	0.71	*b*
	100	5	52, 50	51.0	1.41	*nb*
	90	5	48, 49.5	48.8	1.06	*w*

Simplex 1-2 Worksheet

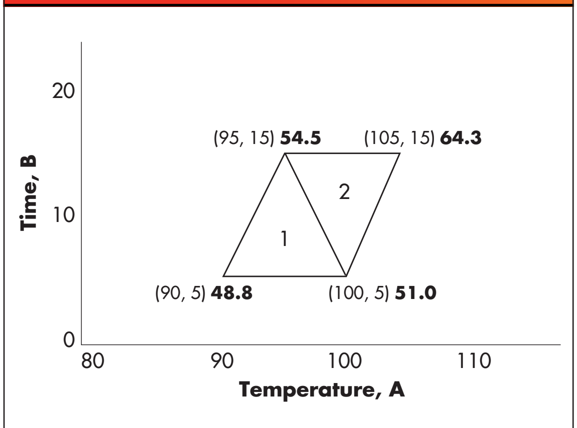
	Factor A	Factor B	Responses (bond strength)	Average $\overline{X}$	Standard deviation S	Rank
Coordinates	95	15	55, 54	54.5	0.71	*b*
	100	5	52, 50	51.0	1.41	*nb*
Σ	195	20				
$\overline{P}=\Sigma/k$	97.5	10				
w	90	5	48, 49.5	48.8	1.06	*w*
$(\overline{P}-w)$	7.5	5				
$R=\overline{P}+(\overline{P}-w)$	105	15	63, 65.5	64.3	1.77	R

$$t_{calc} = \frac{\overline{X}best - \overline{X}worst}{\sqrt{S_p^2\left(\frac{1}{n_{best}} - \frac{1}{n_{worst}}\right)}}$$

For the unique case in which two observations are made for each experimental condition, this equation can be simplified.

$$t_{calc} = \frac{\overline{X}best - \overline{X}worst}{\sqrt{S^2}} = \frac{54.5 - 48.8}{\sqrt{0.814}} = 6.32$$

Plot of Simplexes 1 and 2



Decision rule:

If $t_{calc} > t_{critical}$, then reject H_o and accept H_a.

$t_{critical} = t_{.05,\,2} = 2.92$

For us, 6.32 > 2.92.

Therefore, the difference is significant at 95-percent confidence.

We now determine the coordinates for the next evaluation. Referring to the simplex 1-2 worksheet at the bottom of the preceding page, we'll calculate the centroid for the line formed by coordinates *b* and *nb* hyperface, $\overline{P}$.

Centroid calculations:

- Factor A average coordinates for *b* and *nb* = (95 + 100)/2 = 97.5
- Factor B average coordinates for *b* and *nb* = (15 + 5)/2 = 10
- The centroid, $\overline{P}$, for the hyperface is (97.5, 10)

We extend a line from *w* through $\overline{P}$ by a magnitude equal to the distance between *w* and $\overline{P}$:

- For each factor, the new coordinates are defined by $R = \overline{P} + (\overline{P} - w)$
- For factor A: 97.5 + (97.5 – 90) = 105
- For factor B: 10 + (10 – 5) = 15

This new vertex is called the reflected, or R, vertex because it's a reflection of the line from *w* to $\overline{P}$. The coordinates are (105, 15). We now run an experiment using the values represented by this vertex and obtain two responses. Again, we record the individual responses, the average and standard deviation, and label this vertex as R. Our responses are 63 and 65.5. The plot for simplexes 1 and 2 can be seen in the figure at the left.

After completing the initial simplex 1-2, the following rules apply for creating the next simplexes:

- Never transfer the current row labeled *w* to the next worksheet. The *w* row can be considered the "*waste*" basket.
- The current row *nb* is always designated as *w* on the next worksheet

We now develop the simplex 2-3.

Referring to simplex worksheet 1-2 rows *b*, *nb*, *w* and R:

- Don't use the *w* row from worksheet 1-2 on this sheet.
- Transfer the *nb* row from worksheet 1-2 to worksheet 2-3, relabeling it *w*.
- Rank the remaining two rows and relabel them *b* and *nb* according to their response values

The reflected coordinates have been calculated but we must test for statistical significance between the *b* and *w* vertexes before applying the R vertex.

Test for a statistically significance between the "best" and "worst" responses.

H_o: $U_{best} = U_{worst}$
H_a: $U_{best} > U_{worst}$

$$t_{calc} = \frac{\overline{X}best - \overline{X}worst}{\sqrt{\overline{S^2}}} = \frac{64.3 - 51.0}{\sqrt{2.56}} = 8.32$$

$t_{critical} = 2.92$

8.32 > 2.92; therefore, we reject H_o and accept H_a.

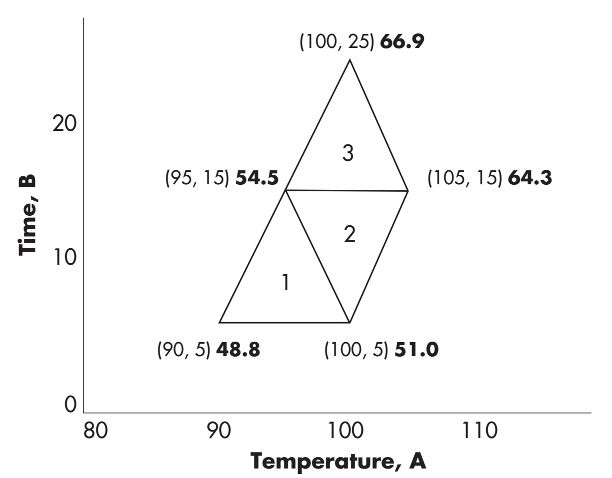
Simplex 2-3 Worksheet

	Factor A	Factor B	Responses (bond strength)	Average $\overline{X}$	Standard deviation S	Rank
Coordinates	105	15	63, 65.5	64.3	1.77	*b*
	95	15	55, 54	54.5	0.71	*nb*
Σ	200	30				
$\overline{P} = \Sigma/k$	100	15				
w	100	5	52, 50	51	1.41	*w*
$(\overline{P} - w)$	0	10				
$R = \overline{P} + (\overline{P} - w)$	100	25	66, 67.7	66.9	1.20	R

Simplex 3-4 Worksheet

	Factor A	Factor B	Responses (bond strength)	Average $\overline{X}$	Standard deviation S	Rank
Coordinates	100	25	66, 67.7	66.9	1.20	*b*
	105	15	63, 65.5	64.3	1.77	*nb*
Σ	205	40				
$\overline{P} = \Sigma/k$	102.5	20				
w	95	15	55, 54	54.5	0.71	*w*
$(\overline{P} - w)$	7.5	5				
$R = \overline{P} + (\overline{P} - w)$	110	25	70, 70.9	70.5	0.64	R

Plot of Simplexes 1,2 and 3



We can now apply the reflected vertex for the simplex 2-3 worksheet. The plot for simplexes 1, 2 and 3 can be seen in the figure above.

Referring to the simplex 2-3 worksheet, we can discard row *w*, relabel row *nb* as *w*, perform the experiment indicated by the R coordinates and rank the remaining two vertexes. Label this worksheet as simplex 3-4. The simplex 2-3 worksheet and the simplex 3-4 worksheet can be found on the preceding page.

Determine if the difference between *b* and *w* is statistically significant. If they are, we can plot the vertex R and complete our plot of simplex 4:

H_o: $U_{best} = U_{worst}$

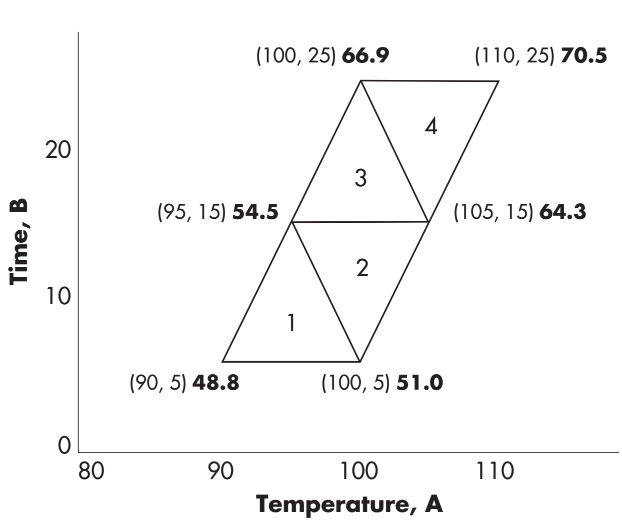
H_a: $U_{best} > U_{worst}$

$$t_{calc} = \frac{\overline{X}best - \overline{X}worst}{\sqrt{S^2}} = \frac{66.9 - 54.5}{\sqrt{0.97}} = 12.6$$

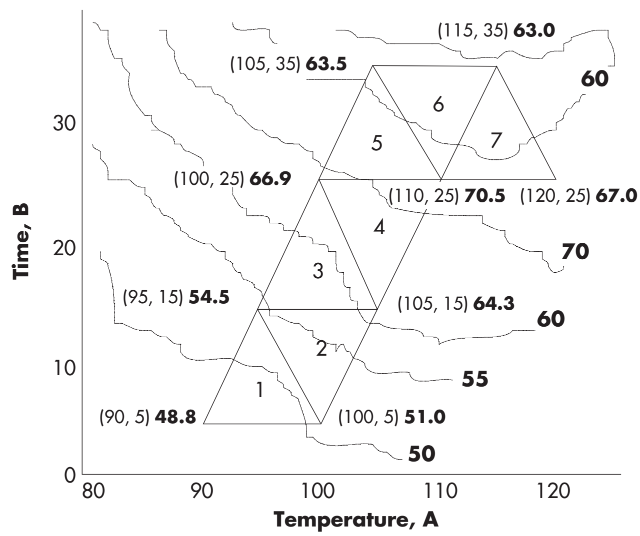
$t_{critical} = 2.92$

12.6 > 2.92; therefore, we reject H_o and accept H_a.

Plot of Simplexes 1, 2, 3 and 4



Plot of Seven Simplexes With Contour for Response Space



We can now apply the reflected vertex for simplex 4. The plot for simplexes 1, 2, 3 and 4 can be seen in the lower figure on the preceding page.

Complete this process for simplexes 4-5, 5-6 and 6-7. Assume that all the *b* to *w* vertexes are statistically significant. The entire plot for all seven simplexes and a contour for the response space can be seen in the figure above.

The limit of possible improvement efforts are reached when simplexes simply revolve around a point. In our hypothetical case, if we created two more simplexes we'd see that they rotate around point 110, 25.

It's possible to overshoot the coordinates that would provide an improvement. This can happen when the size of the original simplex is large, thus giving a lower resolution of the response space. There are techniques for utilizing variable size reflections that allow contracted and expanded reflections. These methods are discussed in *Sequential Simplex Optimization*.

About the author

Mark L. Crossley is president and principle consultant of Quality Management Associates Inc., providing consulting in statistical methods for quality improvement. He received a master's degree in quality assurance from California State University and a bachelor's degree in chemistry/mathematics. He is certified by the American Society for Quality as a CQE, CRE, CQA, CQMgr. and CSSBB. He is also a Master Black Belt and is the author of The Desk Reference of Statistical Quality Methods *(ASQ Quality Press, 2000). Crossley can be reached at (704) 637-2299 or via e-mail at* mcrossley@qualman.com.

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